Effect of Losses on the Spectral Transition of Modal Poles between the Improper and the Proper Riemann Sheets

A.H. Kamel[1] and A.S. Omar[2]

[1]RITSEC, Cairo, Egypt, [2]University of Magdeburg, Magdeburg, Germany

Abstract — Descriptive analysis of the spectral creation and transition of modal poles between the two Riemann surfaces of the γ-plane (γ is the propagation constant) for both lossless and lossy cases is presented. Proper poles are shown to be the spectral continuation of improper ones as a result of crossing between the corresponding Riemann sheets. In contrast to the lossless case, in which the pole transition takes place through the branch points only, poles of the lossy case can cross the branch cuts everywhere. It is also shown that the spectral band width of forward-wave and backward-wave propagation is influenced by losses. Although the presented analysis deals with the simple dielectric slab guide, the obtained results can be (at least qualitatively) generalized to other open guided-wave and leaky-wave structures. Numerical results are presented for lossless, lightly lossy and heavily lossy cases.

SUMMARY

The modal spectrum of open guided-/leaky-wave structures consists of a continuous part resulting from the branch cut integral in addition to one or more types of modes which can be classified into proper guided-wave, improper guided-wave, leaky-wave and reciprocal leaky-wave modes [1] - [5]. For the dielectric slab guide shown in Fig. 1, if a propagation in z-direction according to exp(-γz) is assumed, the γ-plane will have the Riemann sheets and the corresponding branch points and branch cuts shown in Fig. 2 (see e.g. [6]).

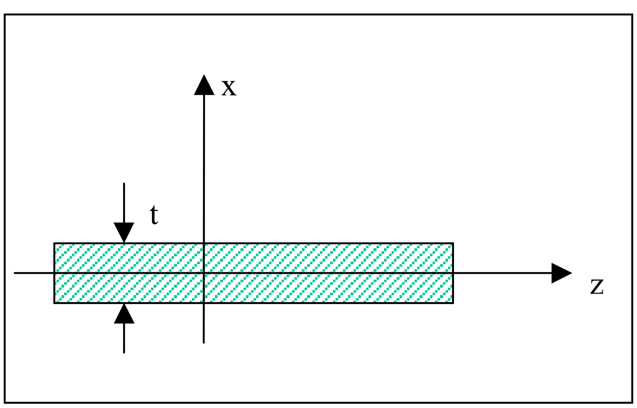

Fig. 1: Dielectric slab guide.

Modal poles on the first and fourth (second and third) quadrants of either sheets are relevant to the propagation in positive (negative) z-direction. We'll restrict our discussion to the propagation in positive z-direction only. For the lossless case proper (improper) guided-wave modes are characterized by a finite number of modal poles on the positive jβ-axis of the proper (improper) sheet above the branch point jk_o. Leaky-wave and reciprocal leaky-wave modes correspond to an infinite number of modal poles on the first and fourth quadrant of the improper sheet, respectively.

In the following a descriptive follow-up of the spectral behavior of the different types of poles mentioned above (as k_o increases from zero to infinity) is given. Except for a possible quasi TEM mode with zero cutoff, all modal poles for $k_o = 0$ are of leaky-wave type. They lie on the improper sheet in doubly symmetrical *quadruplets* (one pole for each quadrant) as shown in Fig. 2. As k_o increases, the nearest pole quadruplet to the origin approaches the imaginary jβ-axis in pairs with P_1 and P_4 (P_2 and P_3) being assigned to the propagation in positive (negative) z-direction (see Fig. 2). As P_1 (P_3) and P_2 (P_4) meet, a double pole PP_u (PP_l) with $\beta_{PPu} > k_o$ ($\beta_{PPl} < -k_o$) is created giving rise to a degenerate mode. At this point P_2 and P_4 exchange their assignments. P_1 and P_2 (P_3 and P_4) are now assigned to the propagation in positive (negative) z-direction. If k_0 further increases, the upper (lower) double pole splits down into the two simple poles P_{pru} and P_{imu} (P_{prl} and P_{iml}) which run downwards (upwards) and upwards (downwards) along the imaginary jβ-axis, respectively, giving rise to two improper guided-wave modes. The pole P_{pru} is very short-life on the improper sheet. The corresponding mode is a backward-wave one as β_{pru} decreases as k_o increases. As P_{pru} reaches the branch point jk_o it emerges there into the proper sheet to become a proper guided-wave pole. As k_o further increases, P_{pru} runs upwards along the imaginary jβ-axis of the proper sheet giving rise to a forward-wave propagation. A similar discussion applies to the lower pole P_{prl} (which contributes in fact to the propagation in negative z-direction). On the other hand, the pole P_{imu} is a long-life one which gives rise to an improper guided-wave mode of a forward-wave type.

Fig. 3 shows a typical dispersion characteristic of the two poles P_{pru} and P_{imu}.

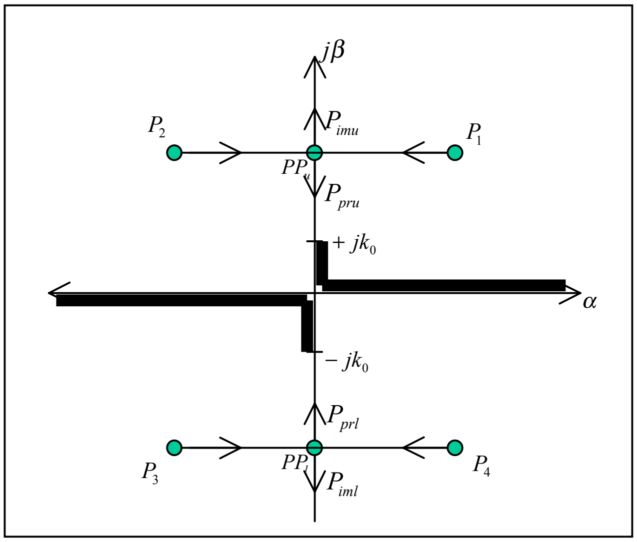

Fig. 2: A pole quadruplet on the improper sheet.

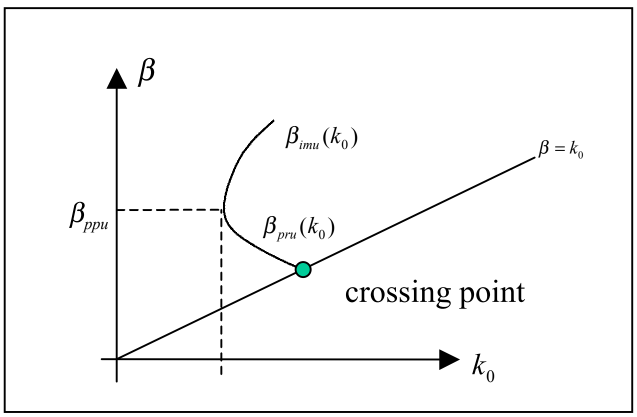

Fig. 3: A typical dispersion characteristic.

If the dielectric slab is lossy, two cases should be distinguished: lightly lossy and heavily lossy. In a lightly lossy case the spectral behavior of a pole quadruplet slightly deviates from that of the lossless case. As k_o increases, P_2 and P_4 approach the vertical part of the RHS and LHS branch cuts, respectively, as shown in Fig. 4. Upon arriving there, they exchange their assignments and emerge to the first and third quadrant of the proper sheet, respectively, where they become of a proper type. On the other hand, P_1 and P_3 move nearly parallel to the imaginary $j\beta$-axis upwards and downwards, respectively. Due to the losses, no guided-wave modes are possible in the strict sense (i.e., with $\gamma = j\beta$). Both P_1 on the first quadrant of the improper sheet and P_2 after crossing from the second quadrant of the improper sheet to the first quadrant of the proper sheet represent just *quasi* guided-wave modes and correspond to a propagation constant with positive real and imaginary parts (where the positive real part accounts for the losses). Investigating the modal x-dependence (normal to the air-dielectric interface) shows that the proper (improper) quasi guided-wave mode corresponds to an exponentially decaying (increasing) incoming (outgoing) wave. Consequently, light losses influenced the guided-wave modes in such a way that the improper one becomes slightly leaking while the proper one becomes slightly absorbing (in other words, the modal Poynting vector is slightly tilted out of and towards the dielectric slab for improper and proper guided-wave modes, respectively). The light absorption accompanying the proper mode feeds in fact the losses within the dielectric slab.

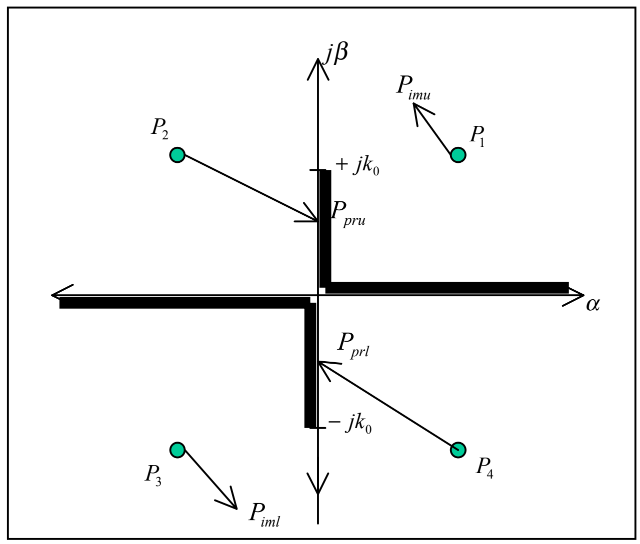

Fig. 4: Typical pole behavior for light losses.

The pole quadruplet of a heavily lossy case behaves spectrally as shown in Fig. 5. While P_1 and P_3 behave more or less similar to that of the lightly lossy case, P_4 and P_2 emerge now (as k_o increases) through the horizontal part of the RHS and LHS branch cuts into the first and third quadrant of the proper sheet, respectively, to become there of a proper type (without exchanging their assignments). All poles lie now far from the imaginary $j\beta$-axis confirming the fact that losses are heavy.

A modal feature which has not been enough investigated in the literature is whether a modal propagation is of forward- or backward-wave type (forward- and backward-wave propagations correspond to increasing and decreasing β as a function of k_o, respectively). We've found out that all improper modes (whether of leaky-wave or guided-wave type) change their nature from forward-wave to backward-wave and vise versa as k_o changes. Losses affect this behavior to some extent, as will be shown in the numerical examples given below.

NUMERICAL EXAMPLES

As an illustrative case we will consider the odd TE modes of the dielectric slab guide shown in Fig. 1. Fig. 6 shows the dispersion characteristics of the lowest order quadruplet of leaky-wave poles (P_1, P_2, P_3, P_4) for the lossless case as k_o increases from zero. The corresponding propagation constants are $\gamma_1 = \alpha_o + j\beta_o$, $\gamma_2 = -\alpha_o + j\beta_o$, $\gamma_3 = -\alpha_o - j\beta_o$, $\gamma_4 = \alpha_o - j\beta_o$, respectively, with positive α_o and β_o. For $0 < k_o < k_{o1}$, β_o decreases as k_o increases. P_1 represents then a leaky-wave mode with a backward-wave nature. For $k_{o1} < k_o < k_{o2}$, β_o becomes an increasing function of k_o and P_1 represents now a leaky-wave mode with a forward-wave nature. At k_{02} a double pole is generated as P_1 and P_2 combine together, which gives rise to a degenerate guided-wave mode (as α_o vanishes now). For $k_{o2} < k_o < k_{oc}$ the degenerate mode is spectrally continued into two improper guided-wave modes. The one with the increasing propagation constant is a long-life one with a forward-wave nature. The one with the decreasing propagation constant is very short-life with a backward-wave nature. It arrives shortly the branch point and crosses there into the proper sheet where it becomes a proper guided-wave mode of a forward-wave type as its propagation constant (dashed curve) increases now with k_o.

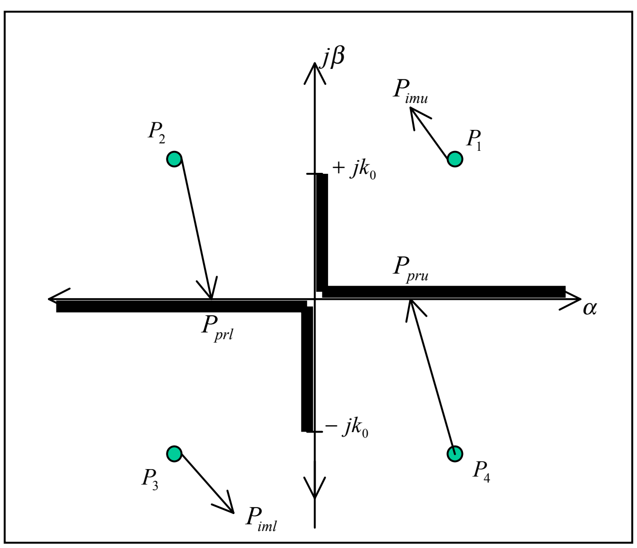

Fig. 5: Typical pole behavior for heavy losses.

Fig. 7 shows the same dispersion characteristics for a lightly lossy structure. The pole quadruplet shows now only simple point symmetry with $\gamma_1 = \alpha_1 + j\beta_1$, $\gamma_2 = -\alpha_2 + j\beta_2$, $\gamma_3 = -\gamma_1$, $\gamma_4 = -\gamma_2$. For $0 < k_o < k_{oc}$, where k_{oc} is the value of k_o at which the pole crossing from the improper into the proper sheet takes place (cutoff), both P_1 and P_4 are assigned to the propagation in positive z-direction. For $k_o > k_{oc}$, P_2 and P_4 have exchanged their assignments after having crossed into the first and third quadrant of the proper sheet, respectively. P_1 and P_2 are now assigned to the propagation in positive z-direction. Again frequency ranges of forward-wave and backward-wave propagation exist. The degeneracy accompanied the lossless case disappeared now.

Fig. 8 describes a heavily lossy structure. The main difference to the above cases is that P_2 and P_4 don't exchange their assignments anymore. P_1 and P_4 remain being assigned to the propagation in positive z-direction for all frequencies. The cutoff of a proper guided-wave mode (or in other words the cross-over of the corresponding pole from the improper to the proper sheet) takes place across the real α-axis ($\beta = 0$). It is also worthy noting that the attenuation constants of the different modes posses characteristic minimums. This should have useful practical applications at least as far as the proper guided-wave mode is concerned.

REFERENCES

[1] T. Tamir and A.A. Oliner, "Guided complex waves. Part 1. Fields at an interface", Proc. IEE, vol. 110, no. 2, 1963, pp. 310-324.

[2] T. Tamir and A.A. Oliner, "Guided complex waves. Part 2. Relation to radiation patterns", Proc. IEE, vol. 110, no. 2, 1963, p. 325.

[3] N. Marcuvitz, "On field representation in terms of leaky modes or eigenmodes", Trans. IRE, 1965, AP-4, p. 192.

[4] T. Tamir, "Inhomogeneous wave types at planar interfaces: II-Surface waves:, Optik 37, 1973, pp. 204-228.

[5] T. Tamir, "Inhomogeneous wave types at planar interfaces: III-Leaky waves:, Optik 38, 1973, pp. 269-297.

[6] R.E. Collin, Field Theory of Guided Waves. McGraw-Hill, 1962.

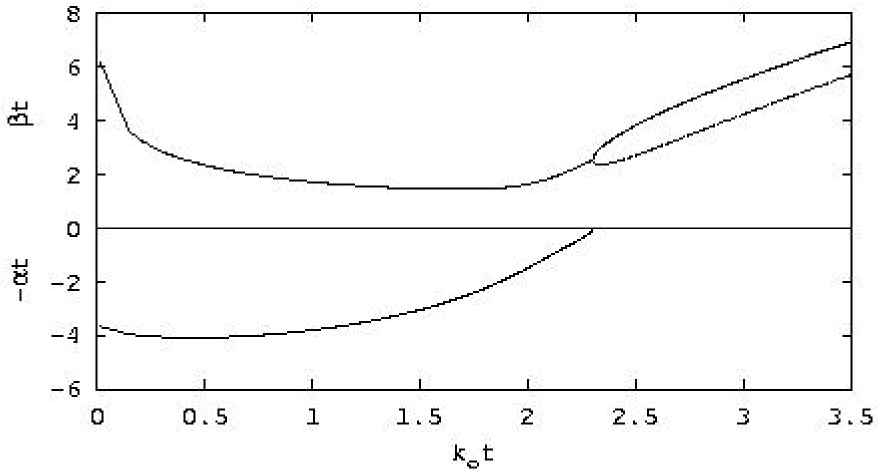

Fig. 6: Dispersion characteristics of a lossless case (positive values for βt, negative values for -αt). Solid lines: pole is always improper, dashed lines: pole changes from improper to proper. $k_{o1}t = 1.67$, $k_{02}t = 2.30$, $k_{oc}t = 2.35$.

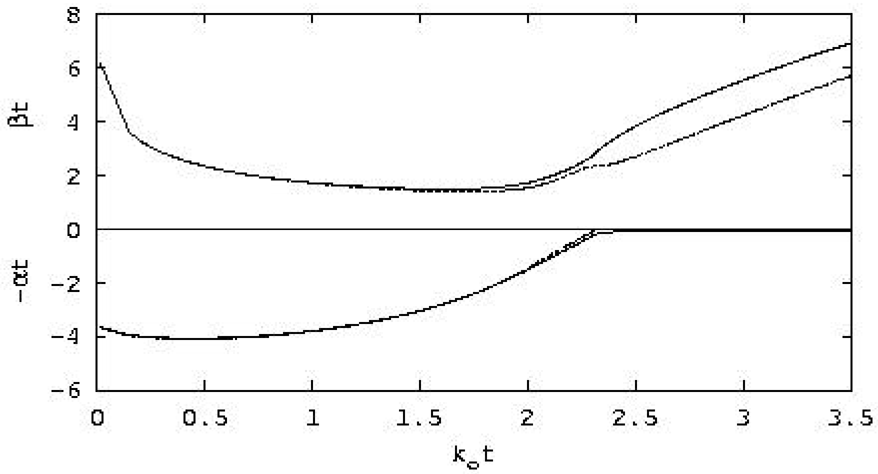

Fig. 7: Dispersion characteristics of a lightly lossy case (positive values for βt, negative values for -αt). Solid lines: pole is always improper, dashed lines: pole changes from improper to proper. $k_{oc}t = 2.35$.

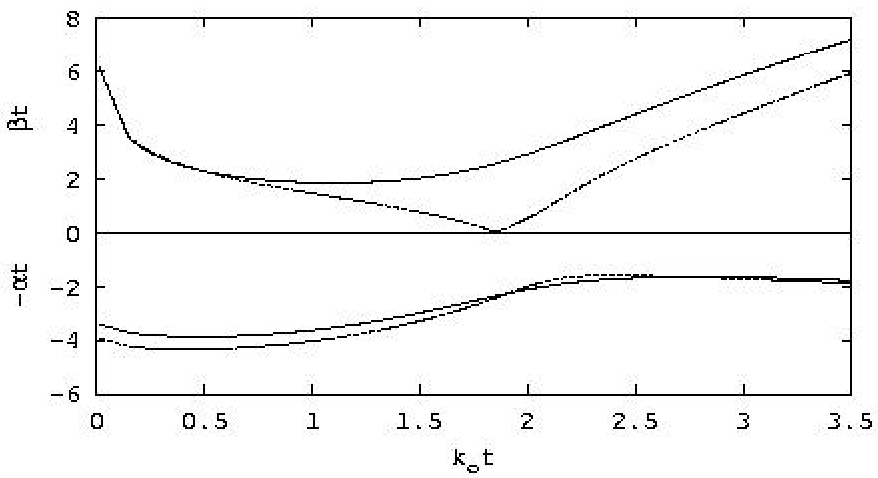

Fig. 8: Dispersion characteristics of a heavily lossy case (positive values for βt, negative values for -αt). Solid lines: pole is always improper, dashed lines: pole changes from improper to proper. $k_{oc}t = 1.85$.